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Crack Development and Its Dynamic Nature of Propagation through Structures: A Study on the Effect of Damped Oscillations of Elastic Plastic Shallow Shell

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Abstract

A simple but efficient method for the analysis of vibrations of elastic plastic shallow shells with damping due to crack generation and its propagation during vibrations is presented. The method is based on the concept of iso-deflection contour lines in conjunction with Ilyushin's method of small elastic-plastic deformation. The governing differential equations are derived and solved with illustration.

Keywords: *Crack Development, Damped Oscillations, Propagation.*

Introduction

Considerable interest has been shown in the past in the analysis of vibration of shallow shells. This interest has been engendered by the widespread use of shallow shells structures in engineering and building design, especially, in several of the present day high technology industries. For example, the industry of nuclear power generation, defense and space material manufacture and modern building industry are industries where shell structure analysis is important due to severe vibration that many modern structures are expected to withstand. However, despite the simplified nature of shallow shell theory and the effect that has been expanded in the area, relatively few solutions are known when the materials behaves plastically. Moreover, in all practical purposes, every vibrating structure is expected to experience some resisting forces resulting in a vibration, damped to some extent. Also, damping becomes obvious when cracks / flaws are generated within the structures due to numerous reasons. In practice, it is very important to note that the materials are not perfectly elastic but those undergo significant plastic deformation at the tip of crack. Moreover, the external load system is restricted in simplified linear theories but for proper estimation of the problem nonlinear analysis is inevitable. It is well known that as and when crack is produced within the materials of the structure degradation of its material properties will start and consequently flexural rigidity (D) will change, which in turn, changes the nonlinear deflections parameters and nonlinear time period of vibrations.

The present paper aims at investigating the dynamic responses of elastic plastic shallow shells with moderate damping caused by crack generation and its propagation through structures using constant deflection contour lines method whose validity and applicability have already been established by several investigators [2-5].

Derivation of the Governing Differential Equations for a Non Cracked Structure and a Cracked Structure

(a) For the case of dynamic responses of a shallow shell of uniform thickness:

Let us consider a shallow shell of thickness 'h' of an elastic-plastic material. Let the equation of the middle surface of the shell referred to a system of orthogonal coordinates (x, y, z) be given by

$$Z = (x^2 / 2R_x) + (xy / R_{xy}) + (y^2 / 2R_y) \qquad (1)$$

where, $r = \sqrt{(x^2 + y^2)}$, is considered small in comparison to the least of the radii of curvature, R_x, R_{xy} and R_y which are taken to constants.

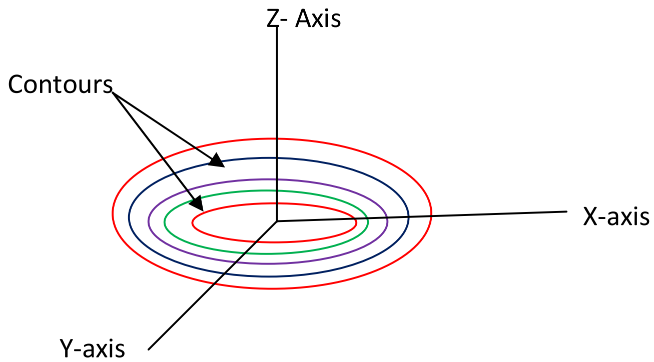

Figure-I: Iso-deflection contour lines with axes system

Figure –II: Cylindrical and spherical shallow domes.

When the shell vibrates in a normal mode, then at any instant t, the intersections between the deflected surface and the parallels z= constant yield contours which after projection onto z=0 surface are the level curves called the 'lines of equal deflection". Let us denote the family of such curves by u(x,y)= constant. For axi-symmetric free vibrations, the intersections of the deflected surface and the parallels z = constant yield contour lines of constant deflection. Application of D` Alembert`s principle to an element of the shell bounded by such a contour at any time τ and subsequent summation of the forces in the direction normal to the surface yields the following dynamical equations [1] :

$$\int V_n \, ds + \int\int [\rho h \, (\partial^2 w) / \partial \tau^2 + (N_x) / R_x + (N_y) / R_y + 2(N_{xy})/R_{xy}] \, dx \, dy = 0 \qquad (2)$$

The transverse reaction forces, $V_n = Q_n - \partial/\partial s(M_{nt})$ (in absence of fractures), (3)

$$V_n = Q_n - \partial/\partial s(M_{nt}) – F(б, l ,Y) \text{ (in presence of crack)} \tag{4}$$

Represents the effect of the shearing force Q_n and the edge-rate of change of twisting moment Mnt along the contour C_u ,

$l = \sum s^2_{i,}$ (all possible crack lengths (s) are to be taken into account),

According to Ilyushin`s theory of the elastic plastic deformation (1948), the bending moments M_x, M_y, M_{xy} and their shear forces Q_x, Q_y are given by the following relations:

$$\begin{aligned} M_x &= - D (1 – v) \{ (\partial 2w/\partial x2) + v (\partial 2w/\partial y2)\} \\ M_y &= - D (1 – v) \{ (\partial 2w/\partial y2) + v (\partial 2w/\partial x2)\} \\ M_{xy} &= D (1 – v) (1 – \Omega) (\partial 2w / \partial x\partial y) \\ Q_x &= (\partial/\partial x) \{ M_y\} – (\partial/\partial y) \{M_{xy}\} \\ Q_y &= (\partial/\partial y) \{ M_x\} – (\partial/\partial x) \{ M_{xy}) \} \\ Q_n &= Q_x \operatorname{Cos}\alpha + Q_y \operatorname{Sin}\alpha \end{aligned} \tag{5}$$

$$M_{nt} = M_{xy} (\operatorname{Cos}2\, \alpha - \operatorname{Sin}2\, \alpha) + (M_x - M_y) \operatorname{Sin} \alpha \operatorname{Cos}\alpha \tag{6}$$

Where, $\operatorname{Cos}\alpha = (dy / ds)$ and $\operatorname{Sin}\alpha = - (dx / ds)$.

Here, ρ, h and w are respectively , the mass density, the shell thickness and the deflection. Using the well known expressions for the moments and shearing forces and assuming that the membrane forces $N_{x,}$, N_y and N_{xy} are given by

$$N_x = (\partial 2 \Phi / \partial y2),\ N_y =(\partial 2 \Phi / \partial x 2),\ N_{xy}= - (\partial 2 \Phi / \partial x \partial y) \tag{7}$$

Equation (2) finally reduces to:

$$\begin{aligned} &(\partial^3 w/\partial u^3) \int(1 - \Omega) R ds + (\partial^2 w/\partial u^2) \int(1 - \Omega) F ds + (\partial w/\partial u) \int(1 - \Omega) G ds \\ &+ (\partial^2 w/\partial u^2) \int D [(\partial \Omega /\partial x) (\partial u /\partial x) + (\partial \Omega /\partial y) (\partial u /\partial y)] \sqrt{t} ds + (\partial w/\partial u) \int (D / \sqrt{t}) [K(\partial \Omega /\partial x) + L \\ &(\partial \Omega /\partial y)] ds + \iint [\rho h (\partial^2 w/\partial \tau^2 (1/ R_x)(\partial^2 \Phi / \partial y^2) + (1 / R_y) (\partial^2 \Phi / \partial x^2) - \\ &2 / (R_{xy} \partial^2 \Phi / \partial x \partial y] dx dy = 0 \end{aligned} \tag{8}$$

Where, $R = - D t^{3/2}$,

$F = - D / (\sqrt{t}) [3 u,_{xx} u,_{x 2} +3 u,_{yy} u,_{y2} + u,_{xx} u,_{y} +u,_{yy} u,_{x} 2\ 4\ u,_{xy} u,_{x} u,_{y}]$

$G = - D / (t)_{1 ..5} [u,_{xxx} u,_{x 3} + u,_{yyy} u,_{y3} + (2 – v) (u,_{xxx} u,_{x} u,_{y 2}+u,_{xy} u,_{y3} + u,_{yyy} u,_{y} u,_{x 2} + u,_{xyy} u,_{x 3}) + (2 v - 1) (u,_{xyy} u,_{x} u,_{y 2} + u,_{xxy} u,_{x 2} u,_{y}) + (1 – v) (u,_{xx} – u,yy) (u,_{xx} u,_{y 2} – u,_{yy} u,_{x 2}) -2 (1 – v) u,_{xy} (u,_{x} u,_{y} u,_{xx} – u,_{y 2} u,_{xy} - u,_{x} 2 u,_{y} + u,_{x} u,_{y} u,_{yy})] + 2 D (1 – v)/ (t)_{2.5} [u,_{xy} (u,_{x 2} – u,_{y2}) – u,_{x} u,_{y} (u,_{xx} –u,_{yy})]_2$

$\sqrt{t} = (u,_{x 2} + u,_{y2})$ and

$D = (E h^3/ 12 (1 – v^2)$, is the flexural rigidity.

Here, $\Omega = 0$ when $e \le 1$, the region is elastic ; when $e > 1$ the region is plastic.

Also, $\Omega = \lambda [1 – (3 / 2e) + (1 /2e_3)$ (9)

And $e^2 = (h^2/3e_s{}^2) [(\partial^2 w/\partial x^2)+(\partial^2 w/\partial y^2)+(\partial^2 w/\partial x\partial y)+(\partial^2 w/\partial x^2) (\partial^2 w/\partial y^2)$ (10)

$= (h^2/3e^2_s) [M (\partial w/\partial u)^2+ N(\partial w/\partial u) (\partial 2w/\partial u2) (\partial w/\partial u) + t^2 (\partial^2 w/\partial u^2)$

in which e_s is the yield strain, v is the poisson`s ratio, D is the flexural rigidity of the plate material, λ is a material constant .

Here , $M =[u ,_{xx 2} + u,_{yy 2} + u,_{xx} u,_{yy} + u,_{xy 2}]$

$N = [2 u,_{x2} u,_{xx} + 2 u,_{y 2} u,_{yy} + u,_{xx} u,_{y 2} +u,_{x} u,_{yy} + 2 u,_{x} u,_{y} u,_{xy}]$

Since the transverse vibration is of prime concern, the effects of the longitudinal and latitudinal inertia terms may be neglected, and one can further assume

$$w = W(x, y)F (t) \tag{11}$$

$$\Phi = \Phi (x, y) F(t) \tag{12}$$

Equation (8) will now reduce to

$\Big[(\partial^3 W/\partial u^3) \int (1 - \Omega)\, R ds + (\partial^2 W/\partial u^2) \int (1 - \Omega)\, F ds + (\partial W/\partial u) \int (1 - \Omega)\, G ds$
$+ (\partial^2 W/\partial u^2) \int D\, [(\partial \Omega/\partial x)(\partial u/\partial x) + (\partial \Omega/\partial y)(\partial u/\partial y)]\, \sqrt{t}\, ds + (\partial W/\partial u) \int (D/\sqrt{t})\, [K(\partial \Omega/\partial x) + L$
$(\partial \Omega/\partial y)\,]\, ds\, \Big]\, F(t) + \iint \Big[\rho h W(t) (\partial^2 F/\partial t^2) + \{ (1/R_x)(\partial^2 \Phi/\partial y^2) + (1/R_y)(\partial^2 \Phi/\partial x^2)$
$- 2/(R_{xy} \partial^2 \Phi/\partial x \partial y \} F(t) \Big]\, dx\, dy = 0$ (13)

Consequently, the condition for continuity of deformation reduces

$\nabla^4 \Phi = \{12D(1 - v^2)\}/h^2 (1 - \Omega)\, [(1/R_x)(\partial^2 w/\partial y^2) + (1/R_y)(\partial^2 w/\partial x^2) - 2(1/R_{xy})(\partial^2 w/$
$\partial x \partial y)$ (14)

This equation must hold over all points in the interior of the shell. After integration over the area and application of Greens theorem one obtains:

$(d^3\Phi/du^3)\int Rds + (d^2\Phi/du^2)\int Fds + (d\Phi/du)\int Gds * -12D^2 (1 - v^2)/h^2 (1 - \Omega)(dW/du)$

$\int [K_x (\partial u/\partial y)^2 + Ky\, (\partial u/\partial x)^2 / t^{½}]\, ds = 0$ (15)

Where K_x and K_y denote curvatures at a point and K xy has been assumed to be zero in accordance with the shallow shell theory. Equations (13) and (14) are now the two basic equations for large amplitude vibration of shallow shell.

Illustration

Let us now consider a clamped dome of non-zero curvature upon an elliptic base. Under symmetry consideration we may write: $u = 1 - x^2/a^2 - y^2/b^2$ (16)

Performing the contour integral taken around the closed contour: $u = 1 - x^2/a^2 - y^2/b^2$ = constant and the double integration extending over the ellipse: $x^2/a^2 + y^2/b^2 = 1 - u$

Equation (13) in non dimensional form becomes

$F(t)(1 - \Omega)(1 - u)(d^3W/du^3) - 2(1 - \Omega)(d^2W/du^2) - (d\Omega/du)[(1 - u)(d^2W/du^2) - 2P\{$
$(1/a^4) + (1/b^4) + 2v/a^2 b^2)(dW/du)] + (\rho h^2 \omega^2 P)/(2 D e_s a^2)(\partial^2 F/\partial t^2) + (dF/dt)[k_d P$
$/(2 D e_s a^2)] - \{(Eh\gamma)/D\}(d\Phi/du)F(t) = 0$ (17)

Where, $P = (a^4 b^4)/(3a^4 + 2a^2 b^2 + 3b^4)$, while equation (14) in non dimensional form will reduce to $(1 - u)(d^3\Phi/du^3) - 2(d^2\Phi/du^2) + (1 - \Omega)\gamma(dW/du) = 0$

(18)

with $\gamma = p(k_x/b^2 + k_y/a^2)$; $W = wh/e_s a^2$; $\Phi = \phi/E e_s a^2$

Method of Solution

On substitution of the value of Ω into equations (17) & (18), one obtains

$[(1-u)(d^3W/du^3) - 2(d^2W/du^2)]\ Q_1 F(t) - [2M(d^2W/du^2)(dW/du) + N(d^2W/du^2)^2 + N(d^3W/du^3)(dW/du) + 2t^2(d^3W/du^3)(dW/du)]\ [(1-u)(d^2W/du^2) - 2P_1(dW/du)]Q_2 F^3(t) - (Eh\gamma/D)F(t)(d\Phi/du) + (\rho h P(\partial^2F/\partial t^2)/(2De_s a^2) + k_d P(dF/dt)/2De_s a^2) = 0$ (19)

and $(1-u)(d^3\Phi/du^3) - 2(d^2\Phi/d^2u) + Q_1\gamma(dW/du) = 0$ (20)

where, $Q_1 = [2e^3(1-\lambda) + \lambda(3e^2-1)]/2e^3$; $Q_2 = (\lambda/4e^5)(e^2-1)a^4$

$P_1 = P(1/a^4 + 1/b^4 + 2\nu/a^2b^2)$

Also, e^2 is given by

$e^2 = ⅓[M(dW/du)^2 + N(dW/du)(d^2W/du^2) + t^2(d^2W/du)^2]$ (21)

Suppose the shell is completely clamped along the boundary. The Clamped edge boundary conditions are given by

$W = 0 = (dW/du)\big|_{u=0}$; $\Phi = 0 = (d\Phi/du)\big|_{u=0}$

With these conditions, equations (19) & (20) are to be solved for W & Φ. However, they do not appear to yield exact solutions and some form of approximation is needed. Here the Galerkin'S method is used. The dependent variables in equations (19) & (20) are approximated by the following trial solutions:

To find an approximate solution, we assume the following trial solutions:

$$W = \Sigma w_j u^j \ ; \quad \Phi = \Sigma \phi_j u^j \qquad (22)$$

Practically the series starts at j=2 as w_1 and ϕ_1 must vanish if the approximate solutions are to satify the boundary conditions. Substitution of the trial solutions in equations (19) & (20) produces the residuals

$$R_1 = [(l-u)\Sigma j(j-1)(j-2)w_j u^{j-3} - 2\Sigma j(j-1) w_j u^{j-2}]Q_1 F(t) - [2M\Sigma j^2(j-1) w_j u^{2J-3} + N\Sigma j^2(j-1) w_j^2 u^{2J-4} + N\Sigma j(j-1)(J-2) w_j^2 u^{2J-4} + 2t^2\Sigma j^2(j-1)(j-2)w_j^2 u^{2J-5}][(1-u)\Sigma j(j-1) w_j u^{j-1} - 2P\Sigma j(j w_j u^{j-1}]Q_1 F^3(t) + (\rho h P(\partial^2F/\partial t^2)/(2De_s a^2) + k_d P(dF/dt)/2De_s a^2) \qquad (23)$$

$$R = [(1-u)\Sigma j(j-1)(j-2)\phi_j u^{j-3} - 2\Sigma j(j-1)\phi_j u^{j-2} + Q_1\Sigma j\phi_j u^{j-1}]F(t) \qquad (24)$$

While e^2 is now given by

$$e^2 = ⅓[M\Sigma j^2 w_j^2 u^{2J-2} + N\Sigma j(j-1) w_j^2 u^{2J-3} + t^2\Sigma j(j-1)^2 w_j^2 u^{2J-4}] \qquad (25)$$

To find the co efficient w_j and ϕ_j for j= 2,3,4........n, we solve the following two sets of equations in accordance with Galerkin's orthogonality procedure (limits of integration from '0' to '1')

$$\int R_1 (u)\, u^j\, du = 0, \qquad j=2,3,4,\ldots\ldots\ldots n \tag{26}$$

$$\int R_2 (u)\, u^j\, du = 0, \qquad j=2,3,4,\ldots\ldots\ldots n \tag{27}$$

It is necessary to find the average value of 'e' over the domain of the ellipse given in equation (12) is to be computed. As demonstration of the solution for the simple case n=2 is substituted into the above equations. The following results yield:

$$(\rho h^2 P \ (\partial^2 F/\partial t^2)\ /\ (6 D e_s a^2) + k_d\ P\ (dF/dt)/(6D e_s a^2) = [\ (4/3)\ Q_1 w_2 + \phi_2\ (Eh\gamma/2D)]\ F(t) + [(4M/5) + (2N/3 - (32MP_1/5) - 4NP_1]\ Q_2\ w^3{}_2\ F^3(t) \tag{28}$$

$$\text{And} \quad \phi_2 = (3/8)\,\gamma\, Q_1\, w_2 \tag{29}$$

While the average value of 'e' happens to be

$$\bar{e} = w_2 \surd\,[\ 1/a^4 + 1/b^4 + 2\ \nu/a^2 b^2\ ]\surd(40/9) \tag{30}$$

From equiations (33) & (34) one obtains the following time differential equation in F(t):

$$F(t) + C_1\ (d^2 F(t)/dt^2) + C_2(dF(t)/dt) + C_3 F^3(t) = 0 \tag{31}$$

where $C_1 = -(\rho h^2 P)/(6 D e_s a^2)\ [\ (4/3)\ Q_1 w_2 + \phi_2\ (Eh\gamma/2D)]$

$$C_2 = -\ k_d\ P\ [\ 4/3 + (3Eh\gamma^2/16D)]\ Q_1\ w_2\ ]/\ [\ (4/3)\ Q_1 w_2 + \phi_2\ (Eh\gamma/2D)]$$

$$C_3 = [\ 4M/5 + 2N/3 - 32\ M\ P/5 - 4NP\ ]\ Q_2\ w^3{}_2]/\ [\ (4/3)\ Q_1 w_2 + \phi_2\ (Eh\gamma/2D)]$$

Since the series is rapidly converging hence considering the first few terms of the series one can obtain the approximate values of the central deflection ' W_0'

$$W_0 = \Sigma\, w_{j,} \qquad [j = 2 \text{ to } n]$$

The solution of equation (31) is given by

$$F(t) = a_0\ e^{-C_1 t}\ \text{Sin}\ [\ C_2\, t\ \{\ 1 + (3/8)\ a_0{}^2 (\ C_3 / C_2\ )\ e^{-C_1 t}\ \} + \theta_0\ ] \tag{32}$$

The time periods of the non-linear and linear oscillations are

$T^* = 2\pi / [\, C_2 \{ 1 + (3/8)\, a_0^{\,2} (C_3 / C_2)\, e^{-C_1 t} \,]$ and $T = 2\pi / C_2$.

Thus $[T^* / T] = [\, 1 + (3/8)\, a_0^{\,2} (C_3 / C_2)\, e^{-C_1 t} \,]^{-1}$, for non- cracked material . (33)

Thus $[T^* / T] = [\, 1 + (3/8)\, a_0^{\,2} (C^*_3 / C^*_2)\, e^{-C_1 t} \,]^{-1}$, for a cracked material (34)

Where $C^*_2 = -(6 D^* e_s a^2) / \rho h^2 P\, [\, 4/3 + (3Eh\, \gamma^2 / 16D^*)]\, Q_1\, w_2$

$C^*_3 = -(6 D^* e_s a^2) / \rho h^2 P\, [\, 4M/5 + 2N/3 - 32\, M\, P/5 - 4NP \,]\, Q_2\, w^3{}_2$

D* is a function of crack length and stress intensity factor during crack generation.

Numerical Results

Numerical results are computed both for elastic and elastic plastic shells based on elliptic planform and these are presented in tables (1 – 2) and through graphs (III-IV). The computations are made with different values of the shallowness parameter ($2\gamma / h$) and material constant $\nu = 0.3$. Moreover, effect of crack are computed with the same equation only changing the term for Vn in equation (4) and making subsequent changes in other equations. It is found that as crack is generated within the material of the structures, degradation of the magnitudes of the elastic constants took place causing a sharp fall in the magnitude of the stress developed within the material body. Consequently, the ratio [T* /T] increases . This is due to the fact that as cracks grow in sizes, elastic reaction falls and the magnitude of stress become smaller and smaller, resulting enhancement of time period which is obvious. It is interesting to note that the behavior of Plastic shell is just reverse that of elastic shells which are reflected in the results shown in figures III and IV.

TABLE- I: Damped vibration of Plastic shallow shells. R = a/b , $2\gamma / h = 0$, $\nu = 0.3$, $\lambda = 1$

R =a/b	Time ↓ W_0 →	0.0	0.5	1.0	1.5	2.0	Ref. [6] 1.0
1.0	t=0 sec	1.0000	1.0256	1.1111	1.2903	1.6666	1.2000
1.0	t=5 sec.	1.0000	1.0204	1.0869	1.2195	1.4706	……
1.0	t = 10 sec.	1.0000	1.0152	1.0638	1.1561	1.3658	……
1.0	t= 20 sec.	1.0000	1.0114	1.0471	1.1126	1.2195	…..
2.0	t=0 sec	1.0000	1.0866	1.4682	3.5398	26.6241	1.4500
2.0	t=5 sec.	1.0000	1.0681	1.3351	2.3474	12.8431	…..
2.0	t = 10 sec.	1.0000	1.0502	1.2365	1.7550	4.2533	…..
2.0	t= 20 sec.	1.0000	1.0372	1.1675	1.4704	2.3403	…..

TABLE- II: Damped vibration of Elastic shallow shells. R = a/b , $2\gamma / h = 0$, $\nu = 0.3$, $\lambda = 1$

R =a/b	Time ↓ W_0 →	0.0	0.5	1.0	1.5	2.0	Ref. [6] 2.0
1.0	t=0 sec	1.0000	0.9582	0.8513	0.7179	0.5887	0.6000
1.0	t=5 sec.	1.0000	0.9662	0.8774	0.7608	0.6415	……
1.0	t = 10 sec.	1.0000	0.9744	0.9052	0.8092	0.7047	……
1.0	t= 20 sec.	1.0000	0.9808	0.9272	0.8498	0.7609	…..

2.0	t=0 sec	1.0000	0.8778	0.0423	0.4436	0.3098	0.3000
2.0	t=5 sec.	1.0000	0.8976	0.6917	0.4994	0.3594	.....
2.0	t = 10 sec.	1.0000	0.9229	0.7496	0.5708	0.4280	.....
2.0	t= 20 sec.	1.0000	0.9402	0.7996	0.6395	0.4894	.....

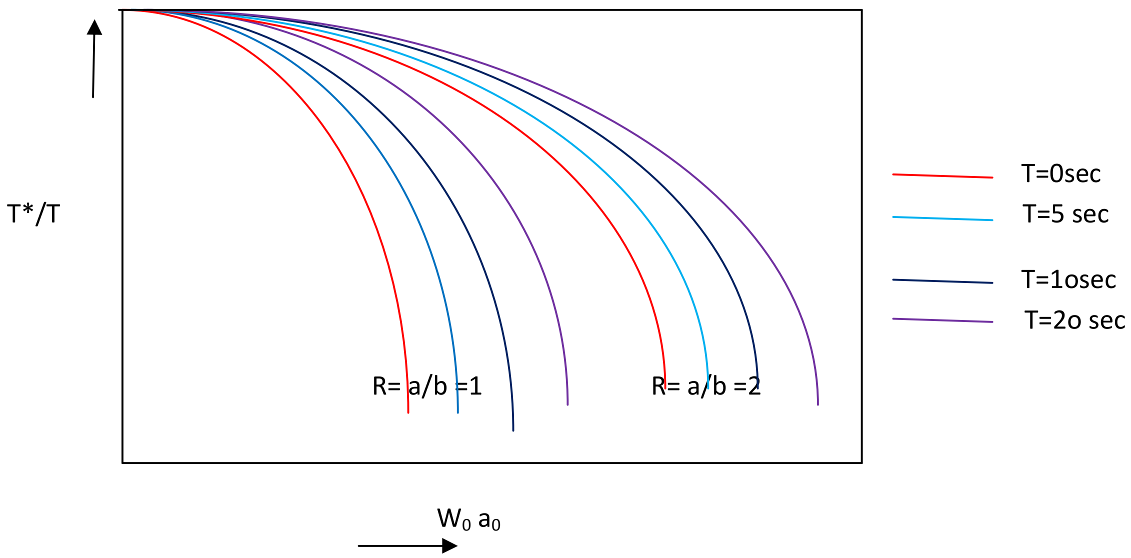

Figure-III: Effect of damping of an elastic shallow shell.

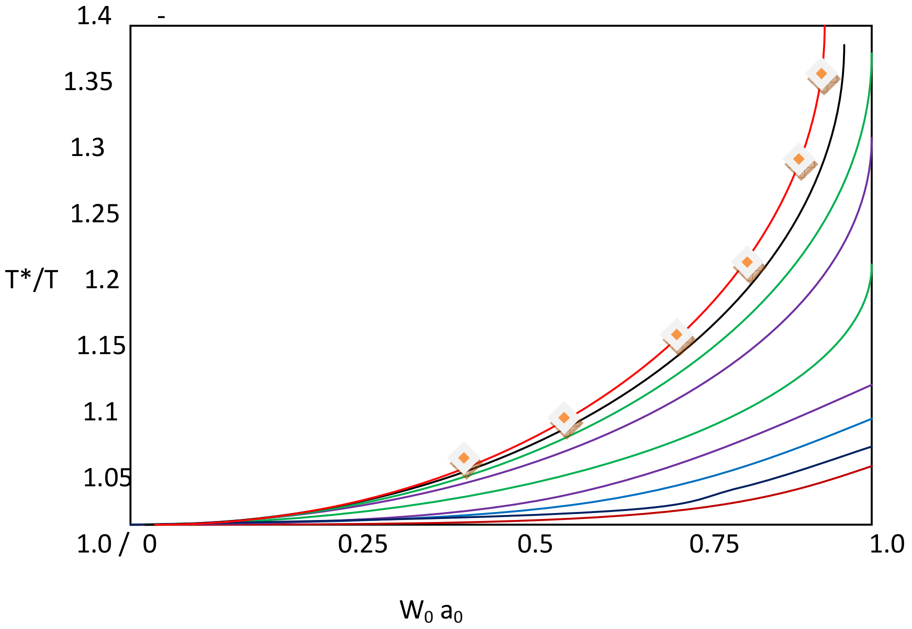

Figure-IV: Effect of damping of a Plastic shallow shell

Observations and Conclusions

It is observed that the results of the present study without exhibiting the effect of damping are in good agreement with those obtained from other methods. The results for damped oscillations seem to be completely new. The results for elastic deformations of circular and elliptic domes are compared with the available results [6]. It is clear from the results that the effect of damping becomes considerably higher and higher as the crack is developed and when it grows in sizes within the material bodies and also as the amplitude of vibrations increases both for elastic and plastic shells. It is also evident that the ratio of the nonlinear to linear time periods approaches to unity as and when the duration of damping force increases. For large amplitude vibrations the velocity of crack propagation increases which in turn enhances the damping force causing the nonlinearity to play a great role.

Though, the elastic –plastic deformation of the shell is analyzed with ease and accuracy by using Ilyushin's theory for small plastic deformation in conjunction with the method of constant deflection contour lines, still the method heavily relies on the accuracy in the choice of the iso-deflection contour function u(x,y). In this study this function is assumed to be the same for corresponding fully elastic case. The main advantage of this method lies in the fact that once the deflection contour function u(x,y) is chosen suitably, the remaining task is very straightforward and only by knowing the shape of the deflection function one can easily solve problem with arbitrary shaped boundary.

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